1 Math 1 HW #1

1. In this problem we calculate Fourier series for two different 2π-periodic functions. Each will give us an infinite series for calculating π.

   (a) Let f be the 2π-periodic function defined by $f(x) = x$ for $-\pi \leq x \leq \pi$. Calculate the Fourier sine series for f. Evaluate this series at $x = \pi/2$ to obtain the approximation

   $$\pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \ldots$$

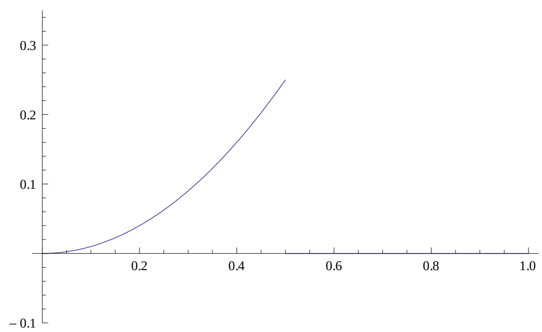
   (b) Let g be the 2π-periodic function defined by $g(x) = |x|$ for $-\pi \leq x \leq \pi$. Calculate the Fourier cosine series for g. Evaluate this series at $x = 0$ to obtain the approximation

   $$\pi = \sqrt{8\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \ldots\right)}$$

   (c) How many terms do you need in each series to get a correct 4-decimal place approximation?

2. Calculate a complex exponential Fourier series for the 1-periodic function defined by

   $$h(x) = \begin{cases} x^2 & \text{if } 0 \leq x < \frac{1}{2} \\ 0 & \text{if } \frac{1}{2} \leq x < 1 \end{cases}$$



3. (a) Define $f(x)$ to be the 1-periodic function given by $f(x) = x^{-1/4}$ for $0 \leq x < 1$. Calculate $\|f\|_2$.

   (b) For a real number c, define define $f_c(x)$ to be the 1-periodic function given by $f_c(x) = x^c$ for $0 \leq x < 1$. For what values of c is $\|f_c\|_2 < \infty$?